Specific Power

The PhD research conducted by Dowdle [2] discusses the design of a high-specific-power, air-cooled electrical machine for aerospace propulsion applications. Dowdle presents a technical model that relates the power per unit volume of a simplified electrical machine to several design parameters:

$$\frac{P}{V} = 2(\bar{\tau})^{\frac{3}{2}}\sqrt{\frac{2\pi\left(\frac{\ell}{r}\right)U^3}{P}}$$

Where $\bar{\tau}$ is the electromagnetic shear stress, ℓ/r is the length-to-radius ratio of the machine, U is the rotor tip speed, and P is the rated power. Assuming that the mass density of the electrical machine $\rho_{EM} = \frac{m}{V}$ is approximately constant across machines, this gives a formula for our primary figure of merit, specific power, as a function of design parameters:

$$SP = \frac{2(\bar{\tau})^{\frac{3}{2}}}{\rho_{EM}}\sqrt{\frac{2\pi\left(\frac{\ell}{r}\right)U^3}{P}}$$

This reveals that, on a first order basis, specific power scales with rated power to the negative one-half power. Dowdle proposes the design of a 3.6 MW electrical machine embedded into the low-pressure compressor of a gas generator. The design has the following parameters:

Specifications [unit]	**Design Value**
Specific Power [kW/kg]	14.8
Electromagnetic Shear Stress [kPa]	55.4
Length-to-Radius Ratio	1.51
Tip Speed [m/s]	230.28
Rated Power [MW]	3.6

The machine mass density is not known exactly, but it can be estimated using our ideal electrical machine specific power formula:

$$\rho_{EM} = \frac{2(\bar{\tau})^{\frac{3}{2}}}{SP}\sqrt{\frac{2\pi\left(\frac{\ell}{r}\right)U^3}{P}}$$

$$\rho_{EM} = \frac{2(55.4e3)^{\frac{3}{2}}}{14.8e3}\sqrt{\frac{2\pi(1.51)(230.28)^3}{3.6e6}} = 9996.42\ \left[\frac{kg}{m^3}\right]$$

This is close to the density of copper (8960 kg/m^3), a major material in electrical machines, so this is likely a reasonable estimate.

Let us consider a sensitivity analysis of the specific power of this design with respect to the electromagnetic shear stress, length-to-radius ratio, and tip speed. Taking partial derivatives of the specific power formula yields:

$$\frac{\partial SP}{\partial \bar{\tau}} = \frac{3(\bar{\tau})^{\frac{1}{2}}}{\rho_{EM}} \sqrt{\frac{2\pi \left(\frac{\ell}{r}\right) U^3}{P}}$$

$$\frac{\partial SP}{\partial \left(\frac{\ell}{r}\right)} = \frac{(\bar{\tau})^{\frac{3}{2}}}{\rho_{EM}} \sqrt{\frac{2\pi U^3}{P\left(\frac{\ell}{r}\right)}}$$

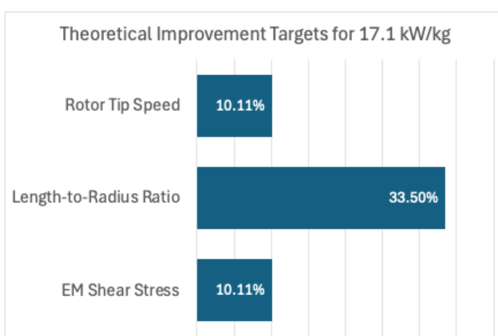
$$\frac{\partial SP}{\partial U} = \frac{3(\bar{\tau})^{\frac{3}{2}}}{\rho_{EM}} \sqrt{\frac{2\pi \left(\frac{\ell}{r}\right) U}{P}}$$

And therefore, the following normalized sensitivities:

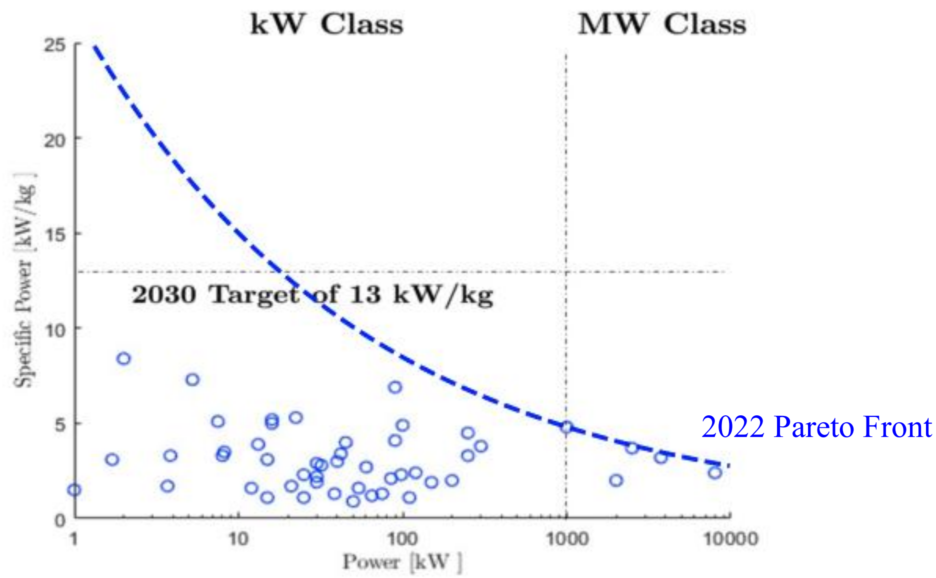
Parameter	Normalized Sensitivity
$\bar{\tau}$	1.5
ℓ/r	0.5
U	1.5

This can be interpreted as the percent increase in each parameter required per percent increase in specific power.

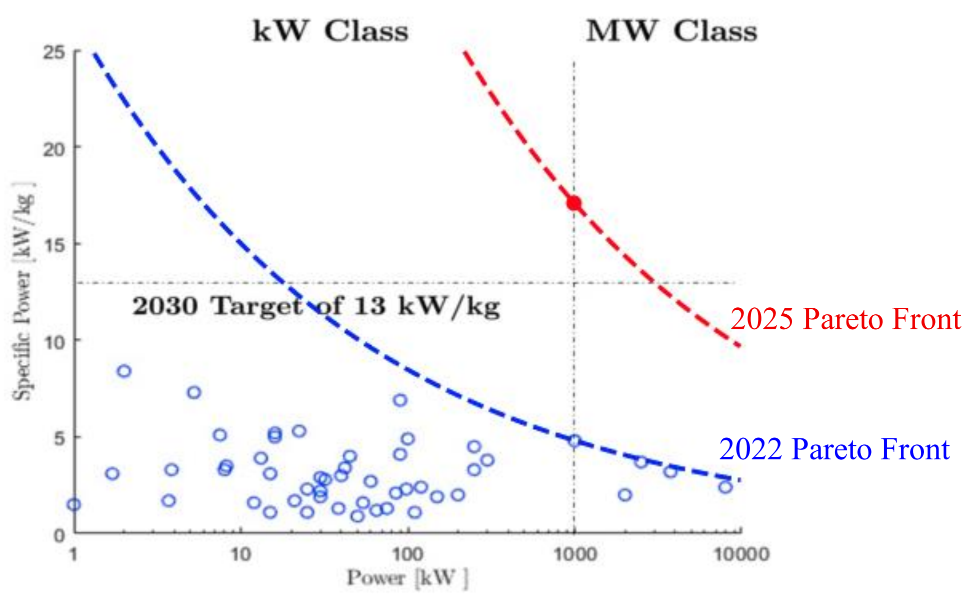
Suppose we wanted to achieve a specific power of 17.1 kW/kg with this 3.6MW machine, the same achieved by the MIT GTL 1MW machine. Using the idealized specific power formula, the required increases to design parameters are given in the following tornado chart:



The major trade space in the design of an electrical machine is between power and specific power. The idealized specific power formula asserts that $SP \sim P^{-\frac{1}{2}}$. However, this neglects the fact that the optimal or achievable EM shear stress, length-to-radius ratio, and tip speed change with the power of the machine. In a sequence of optimizations performed by Dowdle, it was determined that a more realistic scaling law is $SP \sim P^{-\frac{1}{4}}$, which is more favorable for creating high-specific power, high-power machines. This scaling law effectively describes the shape of a praetor front in the power-specific power trade space. The pareto front can be placed on Dowdle's survey of currently available electrical machines:



The achievement of a 1 MW machine with a specific power of 17.1 kW/kg by the MIT GTL in 2025 can be seen as a shift in this pareto front:



Total System Mass

The total system mass of an integrated motor or generator system is the sum of the masses of the electrical machine, power electronics, gearbox, and shaft:

$$m_{IMG} = m_{EM} + m_{PE} + m_{GB} + m_s$$

As shown previously, the state-of-the-art pareto front for electrical machines gives the specific power as a function of the rated power:

$$SP\left[\frac{kW}{kg}\right] = 96.16 \cdot P[kW]^{-\frac{1}{4}}$$

Therefore, given the definition of specific power, we have:

$$m_{EM}[kg] = \frac{P[kW]^{\frac{5}{4}}}{96.16}$$

For the specific power of power electronics, the 2030 goal set by NASA for the STARC-ABL electrified aircraft design is 7.5 kW/kg [2]. Therefore:

$$m_{PE}[kg] = \frac{P[kW]}{7.5}$$

To conduct mission analysis on a hybrid-electric aircraft, Antcliff et al. [3] constructed a regression model that relates gearbox weight to power. Assuming a gear ratio of unity and a power input in kW, the regression model gives:

$$m_{GB}[kg] = 0.453592\left(-37.4262 + 116.3297\left(1.34102 \cdot \frac{P[kW]}{RPM}\right)^{\frac{3}{4}}\right)$$

The amount of material necessary for the shaft is set by the torque on the shaft. The required shaft diameter for a given torque T is given by:

$$D = \left(\frac{16T}{\pi\tau_{yield}}\right)^{\frac{1}{3}}$$

Therefore, for a given power, the required shaft diameter is:

$$D = \left(\frac{16000P[kW]}{\pi\tau_{yield}(RPM)}\frac{60}{2\pi}\right)^{\frac{1}{3}}$$

The weight of the shaft as a function of the material density ρ and an assumed constant length-to-diameter ratio and safety factor is:

$$m_s = SF \cdot \frac{\rho\pi}{4}\frac{\ell}{D}\left(\frac{16000P[kW]}{\pi\tau_{yield}(RPM)}\frac{60}{2\pi}\right)$$

Therefore, we can assemble a model for the total system mass a function of parameters power $P[kW]$, yield strength-to-weight ratio of the shaft material $\frac{\tau_{yield}}{\rho}$, and RPM.

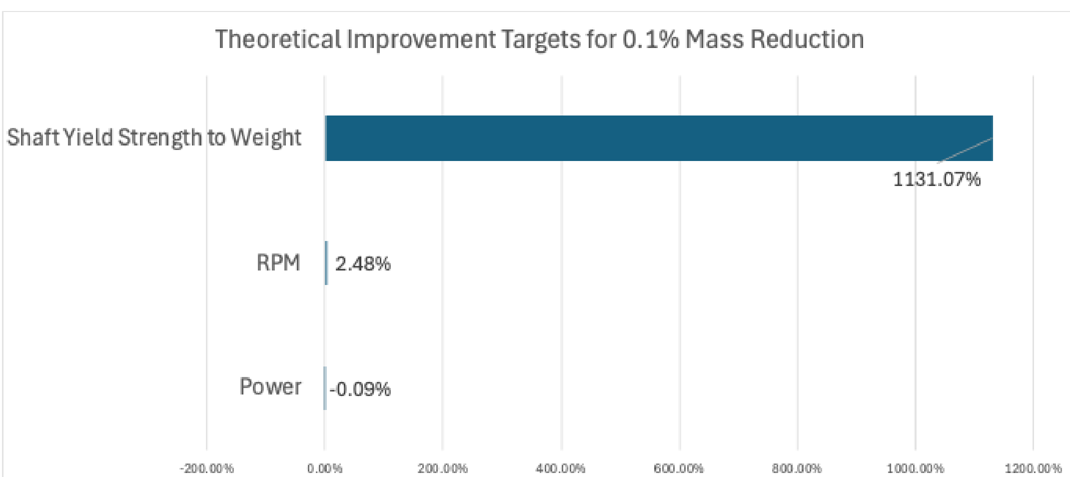
Assuming a safety factor of 2 and a length-to-diameter ratio of 4 for the shaft, the following design parameters are representative of the integration of the MIT GTL 1 MW electrical machine design:

Specifications [unit]	Design Value
Total System Mass [kg]	184.93
Power [kW]	1000
RPM	12500
Yield strength to weight ratio [m^2/s^2]	60727.04

Using finite differences, we can obtain the normalized sensitivities for the parameters:

Parameter	Normalized Sensitivity
P	1.157
RPM	-0.041
τ_{yield}/ρ	-0.001

To achieve just a 0.1% mass reduction, the following improvements to these parameters are required:



This tornado chart is dominated by the lack of sensitivity of mass to shaft structural efficiency. The mass of the system is by far the most sensitive to the power.